cients, B_h^d. Assist can be achieved by making each of the virtual masses m_h^d smaller than its real counterpart in m_h. The effect perceived by the user would be the limb weighing less and showing less inertia. Similarly, a virtual reduction in the damping of the joints can be expected to have an assistive effect on the user.

FIG. **12** illustrates, in schematic form, one way in which the controller for the multi-DOF exoskeleton produces the virtual modification in the properties of the human limb. The natural dynamics of the human limb are represented by the following equation:

$$I_h(m_h,q)\ddot{q}+[C_h(m_h,q,\dot{q})+B_h]\dot{q}+G_h(m_h,q)=\tau_h \quad (16)$$

In the above equation, vector q is a set of n generalized coordinates (typically joint angles) representing the configuration of the limb in joint space. $I_h(m_h,q)$ is the inertia matrix of the limb, $C_h(q,\dot{q})$ represents the centrifugal and Coriolis terms, and $G_h(m_h,q)$ represents the gravitational forces acting on the limb. B_h is the damping matrix of the limb, and the vector τ_h represents the net muscle torques acting on the limb's joints. The effect of the exoskeleton is replacing the limb's natural dynamics by a set of virtual dynamic terms denoted by the superscript d in block (a) of FIG. **12**:

$$I_h^d\ddot{q}+(C_h^d+B_h^d)\dot{q}+G_h^d=\tau_h^* \quad (17)$$

where

$$I_h^d=I_h(m_h^d,q) \quad (18)$$

$$C_h^d=C_h(m_h^d,\dot{q}) \quad (19)$$

$$G_h^d=G_h(m_h^d,q) \quad (20)$$

One way to produce the virtual impedance of the limb is through the interaction forces F_p (in Cartesian space) between the exoskeleton and the human limb. These forces modify Equation 16 as shown in block (b) of FIG. **12**.

$$I_h(m_h,q)\ddot{q}+[C_h(m_h,q,\dot{q})+B_h]\dot{q}+G_h(m_h,q)=\tau_h^*+J_h^TF_p \quad (21)$$

In this equation, J_h^T is the Jacobian matrix of the human limb. The Jacobian matrix relates the Cartesian velocities $\dot{x}$ of the points where the forces F_p are applied, to the limb joints' angular velocities $\dot{q}$. The last term in the above equation can be replaced by a vector of equivalent torques τ_p in joint space:

$$\tau_p=J_h^TF_p \quad (22)$$

These torques can be measured directly, for instance, by installing torque sensors at the joints. Combining Equations 17, 21, and 22 yields the following equation (with some mass and state terms removed for clarity):

$$(I_h^d-I_h)\ddot{q}+[(C_h^d-C_h)+(B_h^d-B_h)]\dot{q}+(G_h^d-G_h)q=-\tau_p \quad (23)$$

This equation can be expressed in compact form by defining

$$I_e^d=I_h^d-I_h,\ C_e^d=C_h^d-C_h,\ B_e^d=B_h^d-B_h,\ G_e^d=G_h^d-G_h \quad (24)$$

We refer to the above terms as the virtual dynamics of the exoskeleton. Thus,

$$I_e^d\ddot{q}+(C_e^d+B_e^d)\dot{q}+G_e^dq=-\tau_p \quad (25)$$

As expected, the virtual dynamics of the exoskeleton are those of an active system. For the particular case of a virtual damping matrix B_e^d composed of constant terms, the virtual dynamics of the exoskeleton will be active if B_e^d is proven to be negative definite.

Equation 25, shown also in block (d) of FIG. **12**, represents the basic control law for the exoskeleton. As in the case of the 1-DOF exoskeleton, one embodiment of this control law is an impedance controller. In such an impedance controller, given the interaction torque $-\tau_p$ as input, the exoskeleton enforces the kinematic trajectory represented by $\ddot{q}$, $\dot{q}$ and q.

Equation 25 does not represent the real dynamics of the exoskeleton. These are represented instead by the equation in block (c) of FIG. **12**:

$$I_e(m_e,q)\ddot{q}+[C_e(m_e,q,\dot{q})+B_e]\dot{q}+G_e(m_e,q)=\tau_e-\tau_p \quad (26)$$

In the above equation, $I_e(m_e,q)$ is the inertia matrix of the exoskeleton, $C_e(q,\dot{q})$ represents the centrifugal and Coriolis terms, and $G_h(m_h,q)$ represents the gravitational forces acting on the exoskeleton. B_e is the damping matrix of the exoskeleton. Vector τ_e represents the actuators' torques. The controller's task is to replace these dynamics with those from Equation 25. This normally involves the use of state and/or force feedback.

5. Implementation of a Multi-DOF Assistive Controller Based on Active Impedance

One consideration in implementing a control architecture for a multi-DOF exoskeleton is the linearization of the exoskeleton plant, that is, making the dynamic properties of the exoskeleton independent of the inputs to the system. As shown below, linearization can be accomplished through the use of a model of the dynamics of the physical exoskeleton.

FIG. **13** illustrates a diagram of the control architecture for the multi-DOF exoskeleton, in one embodiment. The exoskeleton's control comprises three main stages, each of which has its own feedback loop. The first stage is the active impedance element based on the virtual exoskeleton impedance. This element represents the desired dynamic behavior of the exoskeleton. The output of the active impedance element is a reference kinematic trajectory (comprising angular position, angular velocity, and/or angular acceleration) for each of the exoskeleton's actuators. The second stage is the trajectory-tracking controller. This component has the function of issuing the basic control commands necessary for the actuators to follow the reference trajectory. This control block can contain a proportional (P) or proportional-derivative (PD) controller. The third stage is the linearizing (model-based) controller. In the case of a multi-DOF exoskeleton, gravity and coupling between the links are sources of nonlinear dynamics that make the trajectory-tracking control insufficient. This problem is solved by adding a linearizing control that effectively makes the exoskeleton behave as a linear plant. This control stage combines a model of the exoskeleton's true dynamics with kinematic feedback (typically position and velocity) from the physical exoskeleton.

The controller illustrated in FIG. **13** is designed to perform the task outlined in FIG. **12**. In one embodiment, the first control stage comprises an active impedance element based on equation 25. This element receives the measured interaction torque $-\tau_p$ and generates a reference acceleration trajectory $\ddot{q}_r$. Successive integrations of this term generate a reference velocity $\dot{q}_r$ and a reference position q_r.

The second stage is the trajectory-tracking controller (outer-loop control), for example a PD controller that applies the control law

$$\alpha_c=\ddot{q}_r+K_D\dot{e}_r+K_Pe_r \quad (27)$$

where $\dot{e}_r$ and e_r are, respectively, the velocity error and the position error. K_D and K_P are scalar gain matrices. α_c is the commanded acceleration input to the exoskeleton.

The third stage is a model-based controller that translates the commanded acceleration into torque commands τ_e for the actuators. Linearization of the exoskeleton also takes place at this stage. On the basis of Equation 26, the control law for the third stage is given by

$$\tau_e=\hat{I}_e(m_e,q)\ddot{q}+[\hat{C}_e(m_e,q,\dot{q})+\hat{B}_e]\dot{q}+\hat{G}_e(m_e,q) \quad (28)$$

The terms $\hat{I}_e(q)$, $\hat{C}_e(q,\dot{q})$, $\hat{B}_e$ and $\hat{G}_e(q)$ constitute the model of the exoskeleton's real dynamics. Provided that the model